A New Method for the Computation of Nonlinear Magnetic Fields Due to Coils with Imposed Terminal Voltages

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Combining the treatment of the field nonlinearity by using the polarization fixed point technique with the application of the superposition allows a matrix formulation relating the magnetic induction to the coil fluxes and to the polarizations in the actual nonlinear media. Since the linear medium used in the adopted model has the same permeability at each iteration, the matrices involved are computed only once, before the start of the iterative procedure. The method is illustrated for two-dimensional fields. In the case of a periodic regime produced by coils with sinusoidal terminal voltages, each harmonic of the magnetic field can be analyzed independently. It is shown that the proposed method is convergent.

***Index Terms*—Computational electromagnetics, integral equations, iterative algorithms, nonlinear magnetics.**

I. Introduction

The equation satisfied by the vector potential $\boldsymbol{A}$ has the form

$$\nabla \times \hat{\boldsymbol{F}}(\nabla \times \boldsymbol{A}) = \boldsymbol{J} \tag{1}$$

where $\boldsymbol{J}$ is the electric current density. If the constitutive relation $\boldsymbol{B} \xrightarrow{\hat{F}} \boldsymbol{H} = \hat{\boldsymbol{F}}(\boldsymbol{B})$ is linear, the field produced when the terminal voltages of the coils (therefore the magnetic fluxes) are imposed can easily be determined by using (1). Namely, at a given distribution of $\boldsymbol{J}$, the coil fluxes are calculated with $\boldsymbol{A}$ obtained from (1) and, then, all the resulting field quantities are multiplied by the ratio between the imposed flux and the calculated one. When the function $\hat{\boldsymbol{F}}$ is nonlinear, $\boldsymbol{J}$ in (1) is a nonlinear function of $\boldsymbol{A}$. The nonlinearity can be treated by the Newton-Raphson technique, with the current density being corrected iteratively. In the case the field is due to coils connected to sinusoidal voltage sources, the method of static permeability is often employed, with the permeability corrected at each iteration [1]. A linear medium is adopted at each iteration and the field quantities are updated by using the ratio between the imposed terminal voltage and the one computed. Unfortunately, the convergence is not always assured and the field solution is sinusoidal. The solution accuracy becomes unsatisfactory for pronounced material nonlinearities.

In this work, it is proposed to treat the field nonlinearity by employing the Polarization Fixed Point Method [2]. It is proved that the iterative procedure implemented is convergent. The problem solution is illustrated for two-dimensional systems. Certain matrices remain unchanged for all the iterations.

II. Polarization Method for Imposed Coil Voltages

The medium with a nonlinear relationship $\boldsymbol{H} = \hat{\boldsymbol{F}}(\boldsymbol{B})$ is replaced with a "linear" medium having a relationship in the form

$$\boldsymbol{B} = \mu \boldsymbol{H} + \boldsymbol{I} \tag{2}$$

where μ is a constant permeability and the nonlinearity is transferred to $\boldsymbol{I}$ which is a fictitious magnetic polarization,

$$\boldsymbol{I} = \boldsymbol{B} - \mu \hat{\boldsymbol{F}}(\boldsymbol{B}) = \hat{\boldsymbol{G}}(\boldsymbol{B}) \tag{3}$$

$\mu \equiv 1/\nu$ can be chosen such that $\left\| \hat{\boldsymbol{G}}(\boldsymbol{B}') - \hat{\boldsymbol{G}}(\boldsymbol{B}'') \right\|_\nu \le \theta \left\| \boldsymbol{B}' - \boldsymbol{B}'' \right\|_\nu$, $\forall \boldsymbol{B}', \boldsymbol{B}''$, with $\theta < 1$ and the scalar product defined as $\left\langle \boldsymbol{B}', \boldsymbol{B}'' \right\rangle_\nu = \int_{\Omega_{fe}} \nu \boldsymbol{B}' \cdot \boldsymbol{B}'' d\Omega$, where Ω_{fe} is the region occupied by the nonlinear media. Thus, the function $\hat{\boldsymbol{G}}$ is a contraction [2].

The following iterative procedure is used for the solution of the magnetic field problem:

i) An arbitrary $\boldsymbol{I}^{(0)}$ is chosen. With $\boldsymbol{I}^{(n-1)}$ known, at the n-th iteration one determines the field intensity and the magnetic induction $\boldsymbol{B}^{(n)}$ for a linear medium characterized by (2), the coil fluxes (or terminal voltages) being known.

ii) The polarization is corrected with (3), $\boldsymbol{I}^{(n)} = \hat{\boldsymbol{G}}(\boldsymbol{B}^{(n)})$. When the difference $\left\| \boldsymbol{I}^{(n)} - \boldsymbol{I}^{(n-1)} \right\|_\nu$ becomes sufficiently small, the iterative process is ended.

The function $\boldsymbol{I} \xrightarrow{\hat{B}} \boldsymbol{B} = \hat{\boldsymbol{B}}(\boldsymbol{I})$, defined in step i), satisfies the condition $\left\| \hat{\boldsymbol{B}}(\boldsymbol{I}') - \hat{\boldsymbol{B}}(\boldsymbol{I}'') \right\|_\nu \le \left\| \boldsymbol{I}' - \boldsymbol{I}'' \right\|_\nu$, $\forall \boldsymbol{I}', \boldsymbol{I}''$ [2]. Therefore, $\hat{\boldsymbol{B}}$ is nonexpansive when the coil fluxes (or terminal voltages) are imposed. As a consequence, the proposed iterative procedure i), ii) is convergent.

In the case of a *periodic regime*, the polarization in i) is expanded in a Fourier series [3] and a finite number of harmonics is retained. For each harmonic we compute the corresponding harmonic of the magnetic induction and, then, the resultant time-dependent magnetic induction is obtained by superposition. The approximation due to the truncation of the Fourier series is nonexpansive and, thus, the convergence of the iterative procedure is preserved.

III. Field Computation at Each Iteration for Imposed Coil Voltages (2D Structures)

A free space permeability μ_0 may be chosen in (2) for the whole region [2], with $\hat{\boldsymbol{G}}$ in (3) remaining a contraction. The coils occupy the regions ω_+ and ω_-, carrying the current densities J and $-J$, respectively. The coil magnetic flux due to the current density distribution is calculated with

$$\varphi_J = \frac{\mu_0 \zeta J}{2\pi}\left(\int\limits_{\omega_+}\int\limits_{\omega_+} \ln\frac{1}{R} dS_+ dS_+ - 2\int\limits_{\omega_+}\int\limits_{\omega_-} \ln\frac{1}{R} dS_- dS_+ + \int\limits_{\omega_-}\int\limits_{\omega_-} \ln\frac{1}{R} dS_- dS_- \right) \equiv \zeta J L \tag{4}$$

where R is the distance between the integration points and ζ is the number of coil conductors per unit cross-sectional area.

The region Ω_{fe} – which is now the cross section of the ferromagnetic material – is discretized in N_I elements ω_k, the magnetic polarization $\boldsymbol{I}_k$ being considered constant within ω_k, $k = 1,2,\ldots,N_I$. The coil magnetic flux due to the polarization $\boldsymbol{I}_k$ can be expressed in the form

$$\varphi_{I_k} = -\zeta \boldsymbol{P}_k \cdot \boldsymbol{I}_k \tag{5}$$

with

$$\boldsymbol{P}_k \equiv \frac{1}{2\pi}\left[\boldsymbol{k} \times \left(\int\limits_{\omega_+}\int\limits_{\omega_k} \frac{\boldsymbol{R}}{R^2} dS_k dS_+ - \int\limits_{\omega_-}\int\limits_{\omega_k} \frac{\boldsymbol{R}}{R^2} dS_k dS_- \right)\right] \tag{6}$$

where $\boldsymbol{k}$ is the z-axis unit vector and $\boldsymbol{R}$ is the relative position vector of a point in ω_+ or ω_-, respectively, with respect to a point in the element ω_k. The total magnetic flux of the coil is

$$\varphi = \varphi_J + \sum_{k=1}^{N_I} \varphi_{I_k} = \zeta L J - \zeta \sum_{k=1}^{N_I} \boldsymbol{P}_k \cdot \boldsymbol{I}_k \tag{7}$$

which relates J to the total flux φ and polarizations $\boldsymbol{I}_k$,

$$J = \frac{1}{L\zeta}\varphi + \frac{1}{L}\sum_{k=1}^{N_I} \boldsymbol{P}_k \cdot \boldsymbol{I}_k \tag{8}$$

The average magnetic induction over the element ω_i due to the current density can be determined in the form

$$\widetilde{\boldsymbol{B}}_i^J = -\frac{\mu_0 J}{S_i}\boldsymbol{P}_i \tag{9}$$

S_i being the area of the element ω_i. Substituting (8) in (9) gives

$$\widetilde{\boldsymbol{B}}_i^J = \widetilde{\boldsymbol{B}}_i^\varphi - \mu_0 \frac{1}{LS_i}\boldsymbol{P}_i \sum_{k=1}^{N_I} \boldsymbol{P}_k \cdot \boldsymbol{I}_k \tag{10}$$

where

$$\widetilde{\boldsymbol{B}}_i^\varphi = -\frac{\mu_0 \varphi}{LS_i\zeta}\boldsymbol{P}_i \tag{11}$$

On the other hand, the average magnetic induction over the element ω_i produced by the polarization $\boldsymbol{I}_k$ can be calculated with

$$\widetilde{\boldsymbol{B}}_{i,k}^I = \frac{1}{S_i}\overline{\overline{D}}_{i,k}\boldsymbol{I}_k \tag{12}$$

where

$$\overline{\overline{D}}_{i,k} \equiv -\frac{1}{2\pi}\int\limits_{\partial\omega_i}\int\limits_{\partial\omega_k} (d\boldsymbol{l}_i d\boldsymbol{l}_k) \ln R \tag{13}$$

$(d\boldsymbol{l}_i d\boldsymbol{l}_k)$ being the dyad formed by the differential length vectors of the contours of the elements ω_i and ω_k, respectively. Thus, the average over the element ω_i of the total magnetic induction, $\widetilde{\boldsymbol{B}}_i^J + \sum_{k=1}^{N_I} \widetilde{\boldsymbol{B}}_{i,k}^I$, can be expressed directly in terms of the imposed total magnetic flux φ and polarizations in the form

$$\widetilde{\boldsymbol{B}}_i = \widetilde{\boldsymbol{B}}_i^\varphi - \mu_0 \frac{1}{LS_i}\boldsymbol{P}_i \sum_{k=1}^{N_I} \boldsymbol{P}_k \cdot \boldsymbol{I}_k + \frac{1}{S_i}\sum_{k=1}^{N_I} \overline{\overline{D}}_{i,k}\boldsymbol{I}_k \tag{14}$$

Note: L, $\boldsymbol{P}_k$, $\overline{\overline{D}}_{i,k}$ and $\widetilde{\boldsymbol{B}}_i^\varphi$ are calculated analytically, using simple formulas derived in terms of the shape of the discretization elements and their relative position. This is done only once, before starting the iterations.

For a *periodic regime*, the periodic polarization is expanded in a Fourier series and only *N* harmonics are retained,

$$\boldsymbol{I}(t) \cong \sum_{n=1,3,\ldots,2N-1} \left(\boldsymbol{I}'_n \sin(n\omega t) + \boldsymbol{I}''_n \cos(n\omega t) \right) \tag{15}$$

Equation (14) is valid for each harmonic. The time domain expression of the magnetic induction is obtained as

$$\boldsymbol{B}(t) = \sum_{n=1,3,\ldots,2N-1} \left(\boldsymbol{B}'_n \sin(n\omega t) + \boldsymbol{B}''_n \cos(n\omega t) \right) \tag{16}$$

At each step, the magnetic polarization is corrected with (3). The computation time is substantially reduced by considering at the beginning only the fundamental harmonic and, then, increasing the number of harmonics as required in terms of accuracy [3].

IV. Conclusion

The proposed methodology allows the derivation of a matrix relationship for the determination of the magnetic induction in terms of the coil voltages and magnetic polarizations. The respective matrices remain unmodified along the iterative process. Illustrative examples will be presented in the extended version of the paper.

Acknowledgment

This work was supported in part by MEN–UEFISCDI, under project PN-II-PT-PCCA-2013-4-2225.

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